MPM2D
Examination
Exam A, 2011
Length: 3 hours
(Exam set for 2 hrs. + 1 hr. flex time)

OTTAWA-CARLETON
DISTRICT SCHOOL BOARD

Name : Answer Sheet

Teacher : __________

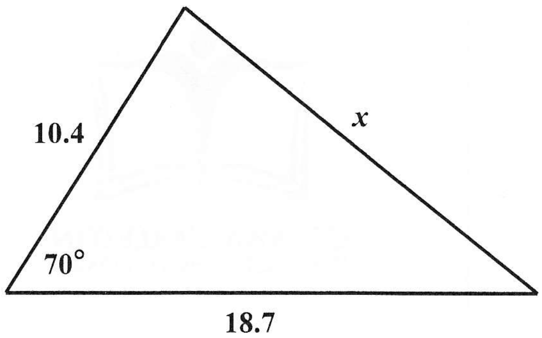
School : __________

Instructions to students:

1. This examination booklet is **15 pages** long. Please check that you have all the pages.
2. Answer all questions with complete solutions in the spaces provided on the examination paper.
3. You may use any school-approved calculator on this examination. Make sure that your calculator is in **DEGREE** mode. Do **not** share your calculator.
4. There is a formula sheet that goes with the examination.
5. Diagrams are not drawn to scale.

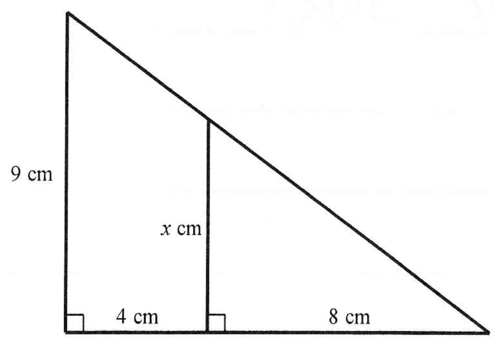
A) Trigonometry

A1) **Solve** for x. Answer to 1 decimal place.



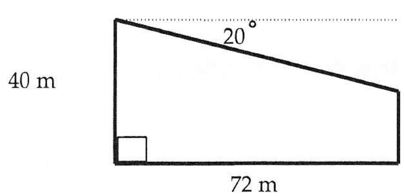
$$x^2 = 10.4^2 + 18.7^2 - 2(10.4)(18.7)\cos 70°$$
$$\doteq 324.8$$
$$\therefore x \doteq 18.0$$

A2) **Solve** for x.



$$\frac{x}{9} = \frac{8}{12}$$
$$\therefore x = 6$$

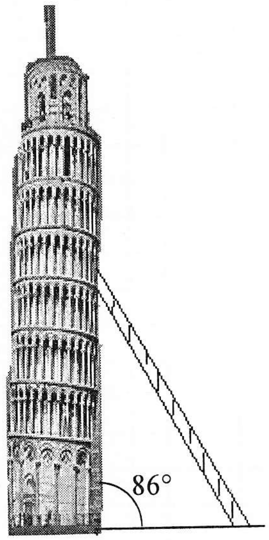
A3) The cost of materials to build a fence (fencing) is \$8.50 per metre. **Determine** the cost of fencing the garden shown below.

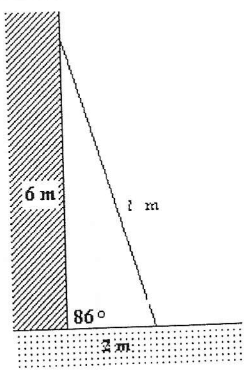


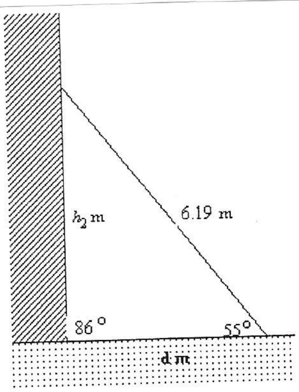
$$\tan 20° = \frac{h}{72} \therefore h \doteq 26.21$$
$$x \doteq 40 - 26.21 \therefore x \doteq 13.79$$
$$\cos 20° = \frac{72}{r} \therefore r \doteq 76.62$$
perimeter $\approx$ 202.4 m

The cost of the fence is approximately \$1720.40

A4) The Leaning Tower of Pisa forms an angle of 86° with the ground.
When it is leaning against the tower, the foot of a ladder is 2 m from the base of the tower, and the top of the ladder is 6m from the base of the tower.
The ladder slips down the tower so that it makes an angle of 55° with the ground.
Does the end on the ground slip more than the end against the tower?
Justify your answer.







First, find length of ladder (l)

$$l^2 = 2^2 + 6^2 - 2(2)(6)\cos 86^\circ$$
$$l = 6.19$$

Next, determine distances after slipping

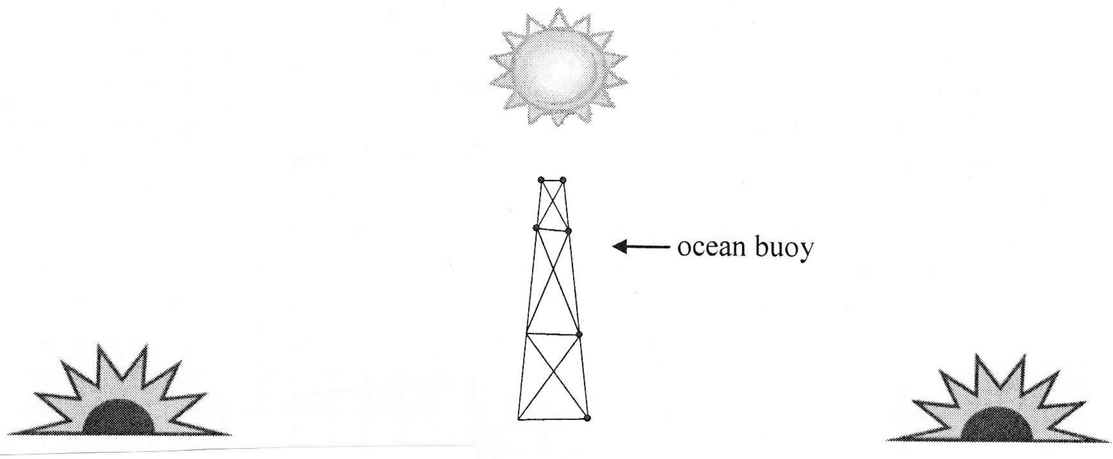
$$\frac{\sin 86^\circ}{6.19} = \frac{\sin 39^\circ}{d} \qquad \frac{\sin 55^\circ}{h_2} = \frac{\sin 86^\circ}{l}$$

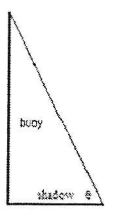
$$d = \frac{(6.19)\sin 39^\circ}{\sin 86^\circ} \qquad h_2 = \frac{(6.19)\sin 55^\circ}{\sin 86^\circ}$$

$$d \doteq 3.90 \qquad h_2 \doteq 5.08$$

So the distance that it slips along the wall is about $6.00\text{ m} - 5.08\text{ m} = 0.92\text{ m}$
and the distance it slips along the ground is about $3.90\text{ m} - 2\text{ m} = 1.90\text{ m}$
$\therefore$ The ladder slips further along the ground.

A5) An ocean buoy is 2.0 m tall. At a certain time of day the shadow of the buoy is 1.0m long.
What possible length(s) will the shadow be 2 hours later?
The time from sunrise to sunset is 15 hours.
Justify your answer.
(**Note:** When the sun is overhead the shadow length is 0 m.)





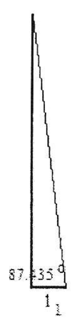
First determine the angle of elevation of the sun, θ

$$\tan\theta = \frac{2}{1}$$

$$\theta = \tan^{-1} 2 \doteq 63.435°$$

From sunrise to sunset sun goes through 180° in the sky in 15 hours or $12°/h$. So in two hours it goes through 24°.

Case 1: It is the morning and the sun is **rising**, so the angle of inclination is increasing. Thus 2 hours later the angle of elevation is about 63.435° + 24° = 87.435°.

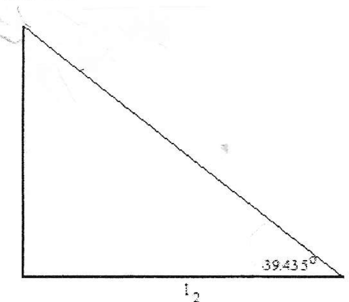


$$\tan 87.435° = \frac{2}{l_1}$$

$$l_1 = \frac{2}{\tan 87.435°}$$

$$l_1 \doteq 0.09$$

Case 2: It is the afternoon and the sun is **setting**, so the angle of inclination is decreasing. Thus 2 hours later the angle of elevation is about 63.435° – 24° = 39.435° and the length of the shadow, l_2, would be:



$$\tan 39.435° = \frac{2}{l_2}$$

$$l_2 = \frac{2}{\tan 39.435°}$$

$$l_1 \doteq 2.43$$

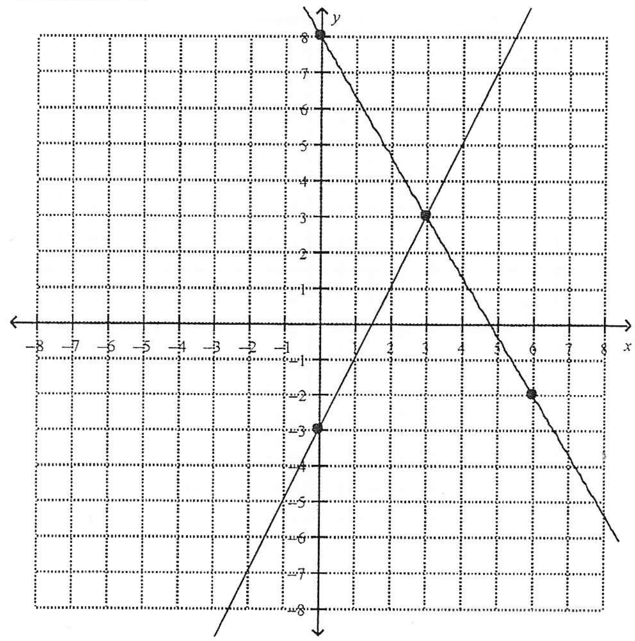
∴ The length of the shadow would be about 0.09 m long if it was the morning and about2.43 m long if it was the afternoon.

B) Analytic Geometry

B1) Two continuous linear relations are defined below:
Determine the point of intersection of the lines defined by these relations.
Justify your solution.

Relation 1:
$y = 2x - 3$

Relation 2:



If we graph the line with equation $y = 2x - 3$ we see that the lines cross at (3, 3)

B2) The school car wash charged \$5 for a car and \$6 for a van.
A total of 86 cars and vans were washed on Saturday and the amount earned was \$475.
Determine the number of vans washed on Saturday.

Let C and V represent the number of cars and vans washed on Saturday, respectively.

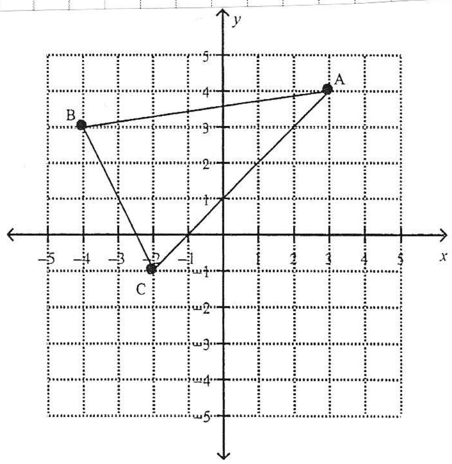
$$C + V = 86 \quad (1)$$
$$5C + 6V = 475 \quad (2)$$
$$5C + 5V = 430 \quad (1)\times 5$$
$$V = 45 \quad (2) - (1)\times 5$$

Substitute $V = 45$ into the first equation

$$C + 45 = 86$$
$$C = 41$$

$\therefore$ They washed 45 vans on Saturday.

B3) Haidar says "if ΔABC is isosceles with AB = AC, then the perpendicular bisector of BC passes through A".
Is Haidar's statement true for the isosceles triangle with vertices (–4, 3), (3, 4) and (–2, –1)?
Justify your answer.



From the diagram AB = AC if A(3, 4), B(–4, 3) and C(–2, –1).

The midpoint of BC is $M\left(\dfrac{-4+(-2)}{2}, \dfrac{3+(-1)}{2}\right) = M(-3,1)$

$m_{BC} = \dfrac{-1-3}{-2-(-4)} \therefore m_{BC} = -2$ .Thus $m_{\perp} = \dfrac{1}{2}$

The equation of the perpendicular bisector of BC is of the form $y = \dfrac{1}{2}x + b$.

Since it goes through M(-3,1) we get $1 = \dfrac{1}{2}(-3) + b \therefore b = \dfrac{5}{2}$,

so the equation of the perpendicular bisector is $y = \dfrac{1}{2}x + \dfrac{5}{2}$.

Substituting $x = 3$ into the equation of the perpendicular bisector we get

$y = \dfrac{1}{2}(3) + \dfrac{5}{2} \therefore y = 4$

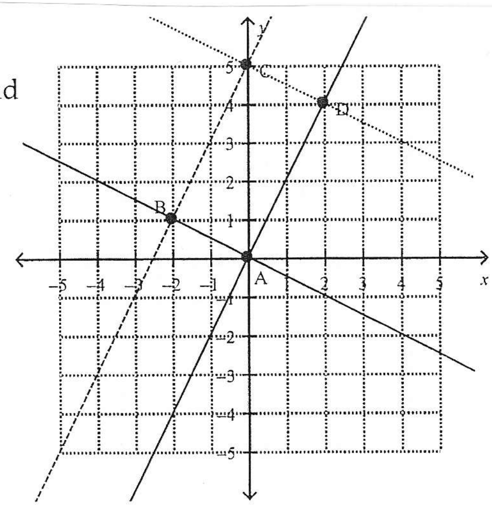
$\therefore$ A(3, 4) is on the perpendicular bisector of BC, so Haidar's statement is true for this triangle.

B4) Given the equation of the line $y = 2x$,

i) State the equations of three other lines so the four lines form a rectangle. Justify your choices.

ii) Determine the area of your rectangle.

Show your work.



The line $y = -\frac{1}{2}x$ is clearly perpendicular to the line $y = 2x$ and it passes though the point B(-2,1).

Thus the line $y = 2x + 5$ passes through (-2, 1) and is parallel to $y = 2x$.

Then we choose the line $y = -\frac{1}{2}x + 5$ as our last line.

Since the enclosed quadrilateral has opposite sides parallel, and adjacent sides perpendicular, it is a rectangle. The lengths of the sides will be

$$d_{AB} = \sqrt{(-2-0)^2 + (1-0)^2}$$
$$= \sqrt{5}$$

and

$$d_{BC} = \sqrt{(0-(-2))^2 + (5-1)^2}$$
$$= \sqrt{20}$$

So

$$A = (\sqrt{5})(\sqrt{20})$$
$$= 10$$

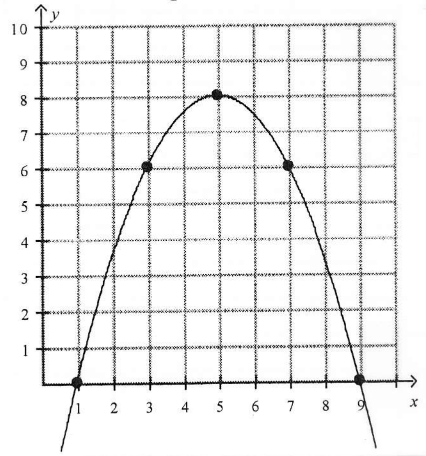
$\therefore$ Thus the area of the rectangle would be 10 square units.

C) Quadratic Relations

C1) **State** the equation of a parabola that opens up and has two zeros. **Justify** your answer.

Answers will vary. For example $y = 5(x+3)^2 - 1$ since $a = 5$ the parabola opens up, and the vertex is (-3, -1) which is below the x axis, so there are 2 zeros.

C2) **Determine** an equation of the parabola graphed below. **Show your work.**



The parabola would have equation $y = a(x-1)(x-9)$.

Find "a" using the point (3, 6) $6 = a(3-1)(3-9) \therefore a = -\frac{1}{2}$

An equation of the parabola is $y = -\frac{1}{2}(x-1)(x-9)$.

The parabola would have equation $y = a(x-5)^2 + 8$.

Find "a" using the point (1, 0) $0 = a(1-5)^2 + 8 \therefore a = -\frac{1}{2}$

An equation of the parabola is $y = -\frac{1}{2}(x-5)^2 + 8$.

Standard form is also acceptable $y = -\frac{1}{2}x^2 + 5x - \frac{9}{2}$

C3) A ball is thrown upward. Its height, h metres after t seconds, is given by the equation $h = -5t^2 + 20t + 1.8$ where 1.8 represents the height at which the ball is released. **Determine** the maximum height of the ball.

Completing the square

$h = -5(t^2 - 4t + 4 - 4) + 1.8$

$h = -5(t-2)^2 + 21.8$

$\therefore$ The maximum height of the ball is 21.8 m.

C4) i) Use a <u>variety</u> of ways to **show** that $x^2 - 2x - 15 = 0$ has two solutions.

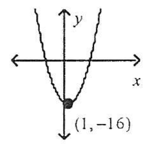
ii) **Explain** how changing one number in the equation $x^2 - 2x - 15 = 0$ can result in no solution. **Justify** your answer.

i) Methods could include:

→ showing that the relation has two zeros: $y = x^2 - 2x - 15$

The equation is equivalent to $y = (x-1)^2 - 16$. This indicates two zeros when $x - 1 = \pm 4$, which corresponds to two solutions for the original equation.

→ showing that the graph of the relation $y = x^2 - 2x - 15 = (x-1)^2 - 16$, which is a parabola which opens up with vertex (1, -16) thus has two x-intercepts. So the original equation has two solutions.



→Solving the equation either by factoring or by using the quadratic formula and showing that there is two solutions:

By factoring:

$$x^2 - 2x - 15 = 0$$
$$(x-5)(x+3) = 0$$
$$\therefore x = -3 \text{ or } x = 5$$

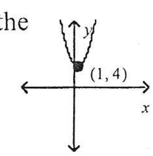
Using the quadratic formula:

$$x^2 - 2x - 15 = 0$$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)}$$
$$x = 5 \text{ or } x = -3$$

→ showing that the discriminant as a positive value: $D = (-2)^2 - 4(1)(-15)$ $= 64 > 0$ *(Note: use of discriminant is not required)*

ii) Students choose a number to modify.

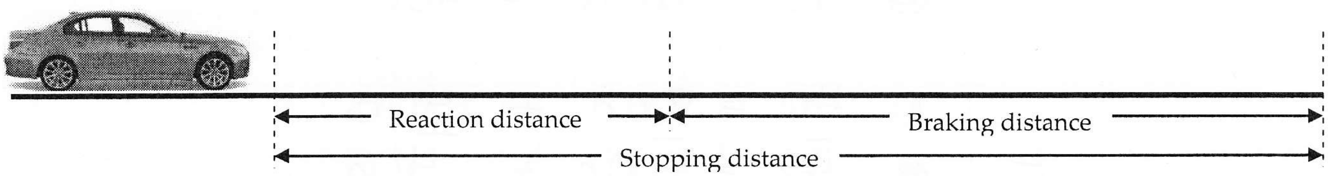
→ Changing the c-value from -15 to 5 would cause the original parabola to move up 20 units resulting in the parabola having no x-intercepts and in having no solutions for the new equation.
When solving for x in the equation $x^2 - 2x + 5 = 0$, the values of x will not be real numbers so there are no solutions.
May generlaize to say that any value for the c- value that is greater than 1 will result in no solutions



→ Changing the b- value from -2 to -4,or 0, or 2 will result in $x^2 - 4x - 15 = (x-2)^2 - 19$, or $x^2 - 15 = 0$, or $y = x^2 + 2x - 15 = (x+1)^2 - 16$. In general, for any b-value, there will be two solutions .

→ Some students might change the a- value, or the zero value on the right side of the equation.

C5) When a driver decides to brake, the total distance required for the car to stop is made up of two parts: The Reaction distance (the distance from when the driver decides to stop to when he/she hits the brakes) and the Braking distance (the distance from when the brakes are hit to when the car stops completely).



The table below gives the distances required to come to a stop from several speeds when Sarah drives her car:

Not important

speed (km/h)	~~Sarah's reaction distance~~ (m)	~~Car's braking distance~~ (m)	Total stopping distance (m)
0	0	0	0
20	16	4	20
40	32	16	48
60	48	36	84
80	64	64	128
100	80	100	180
120	96	144	240

When John drives the <u>same car</u> the relationship between his total stopping distance, d, in metres and his speed, s, in km/h is $d = 0.01s^2 + 1.5s$.

Compare Sarah's and John's total stopping distances if they are traveling at 70 km/h.

Justify your answer.

$y = x^2 + bx + c$ (0,0) y-intercept

$y = x^2 + bx$

sub (20,20)

$y = ax^2 + bx$

$20 = a(20)^2 + b(20)$

① $20 = 400a + 20b$

sub (40,48)

$y = ax^2 + bx$

$48 = a(40)^2 + b(40)$

② $48 = 1600a + 40b$

Solve system

$$① \ (20 = 400a + 20b) \times 2$$
$$② \ 48 = 1600a + 40a$$

$$① \ 40 = 800a + 40b$$
$$- \ ② \ 48 = 1600a + 40a$$

$$\frac{-8}{-800} = \frac{-800a}{-800}$$

$$a = 0.01$$

Sub $a = 0.01$ into ①

$$20 = 400a + 20b$$
$$20 = 400(0.01) + 20b$$
$$20 = 4 + 20b$$
$$20 - 4 = 20b$$
$$\frac{16}{20} = \frac{20b}{20}$$
$$b = 0.8$$

Sarah

$$d = 0.01(70)^2 + 0.8(70)$$
$$d = 105$$

John

$$d = 0.01(70)^2 + 1.5(70)$$
$$= 154$$

So at 70 km/h Sarah take 105m to stop and John takes 154m to stop. Thus John takes 49m more to stop due to slower reaction.